HYDRAULIC TURBOMACHINES
Mock exam – part 2

Duration : 2 hours;

Documentation: personal hand written notes, bilingual dictionary, and available lectures and excercises on the moodle are authorized; you can use your laptop in offline mode (no internet connection).

Exam evaluation: The weight of each question is indicated. General presentation and clarity of answers will be taken into account for the evaluation.

Maximum total score: 30 points

Pumped storage power plant

The outline of a pumped storage power plant is shown in Figure 1. The power plant features 2 units, each one equipped with a pump-turbine. Answer to the questions based on provided values in Table 1. The gravity acceleration, the water density and water kinematic viscosity are g = 9.81 m s^{-2}, ρ = 998 kg m^{-3} and $\nu_{water} = 10^{-6}$ m^2 s^{-1}. In Table 1, the main operating parameters of the power plant are listed.
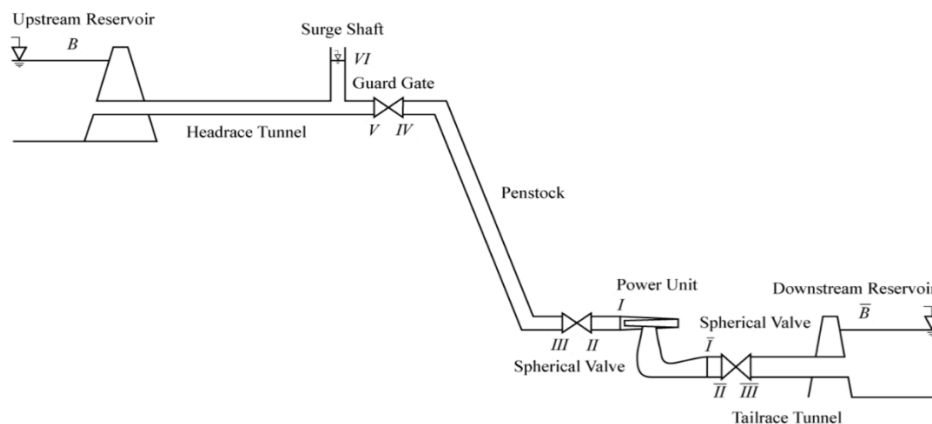

Figure 1 – Outline of the power station

Data	Symbol	Value	Unit
Atmospheric pressure	p_a	1×10^5	Pa
Saturated vapor pressure	p_v	2343	Pa
Headwater reservoir level	Z_B	1250	m
Tailwater reservoir level	$Z_{\bar{B}}$	1025	m
Amount of units in the power plant	$z_{machines}$	2	-
Number of poles pairs	$z_{p,pairs}$	10	-

Frequency of the grid	f_{grid}	50	Hz
Inlet (in turbine mode) section diameter	D_{1e}	3.50	m
Inlet (in turbine mode) section height	B	0.60	m
Outlet (in turbine mode) section diameter	$D_{\bar{1}e}$	2.80	m

Table 1 – Fixed parameters of the power plant

Pumped storage power plant – turbine mode

Energetic efficiency	η_e	0.92	-
Volumetric efficiency	η_q	0.99	-
Global machine efficiency	η	0.90	-
Rated discharge <u>per unit</u>	Q	50	$m^3 s^{-1}$

Table 2 – Turbine mode parameters of the power unit

In a first moment, we consider the power plant in generating mode. The operation parameters are listed in Table 2.

1   Compute the specific potential energy of the installation. [1 point]

$$gH_B - gH_{\bar{B}} = g(Z_B - Z_{\bar{B}}) = 2207.2 \text{ Jkg}^{-1}$$

2   For the rated discharge, the head losses of the installation have been measured and are equal to $\sum gH_r$ = 201.4 J kg^{-1}. Compute the available specific energy of the turbines. Deduce the net head H of the turbines.

$$E = gH_B - gH_{\bar{B}} - \sum gH_r = 2005.8 \text{ J kg}^{-1}$$

$$H = \frac{E}{g} = 204.5 \text{ m}$$

3   Compute the runner frequency n (Hz) and the specific speed v of the runner. [1 point]

The runner frequency is defined by the grid frequency and the amount of poles pairs of the electrical machine:

$$n = \frac{f_{grid}}{z_{p,pairs}} = 5 \text{ Hz}$$

$$\omega = 2\pi n = 31.42 \text{ rad s}^{-1}$$

$$v = \frac{\omega \sqrt{Q}}{\sqrt{\pi} \left(2E\right)^{\frac{3}{4}}} = 0.249$$

4   Compute P_h, the hydraulic power. [1 point]

$$P_h = \rho \, Q \, E = 100.09 \text{ MW}$$

5   Compute the transferred specific energy E_t. [1 point]

$$E_t = \eta_e E = 1845.3 \text{ J kg}^{-1}$$

6   Compute the torque experienced by the runner shaft T_t. [1 point]

We need to consider the leakage flow loss as well:

$$P_t = \omega T_t = \rho \, Q_t \, E_t = \rho \, Q \, \eta_q \, E_t = 91.16 \text{ MW}$$

$$T_t = \frac{P_t}{\omega} = \frac{91.16 \text{ MW}}{31.42 \text{ rad} \cdot \text{s}^{-1}} = 2.9 \times 10^6 \text{ Nm}$$

7   Compute the mechanical efficiency ($\eta_{me} = \eta_{rm} \cdot \eta_m$). [1 point]

$$\eta_{me} = \frac{P}{P_t} = \frac{P_h \cdot \eta}{P_t} = \frac{100.09 \cdot 0.9}{91.16} = 0.988$$

8   Express the Net Positive Suction Specific Energy (NPSE) by $gH_{\bar{I}}$, the machine altitude level Z_{ref}, and the saturated pressure p_v. [1 point]

$$NPSE = gH_{\bar{I}} - \frac{p_v}{\rho} - gZ_{ref}$$

9   Express Thoma number $\sigma = \dfrac{NPSE}{E}$, using the setting level $h_s = Z_{ref} - Z_{\bar{B}}$, the flow velocity $C_{\bar{I}}$, the saturated pressure p_v and the atmospheric pressure p_a. Neglect singular and regular losses inside the tailrace tunnel, and assume that the tailrace tunnel outlet behaves as a water outflow (i.e. a sudden exit). [2 point]

Neglect losses in tailrace tunnel + tunnel outlet as sudden exit:

$$gH_{\bar{I}:-\bar{B}} = \frac{C_{\bar{I}}^2}{2} \rightarrow gH_{\bar{I}} = \frac{p_{\bar{B}}}{\rho} + gZ_{\bar{B}} + \frac{C_{\bar{I}}^2}{2}$$

The Thoma number can be expressed as:

$$\sigma = \frac{NPSE}{E} = \frac{\dfrac{p_a}{\rho} + gZ_{\bar{B}} + \dfrac{C_{\bar{I}}^2}{2} - \dfrac{p_v}{\rho} - gZ_{ref}}{E} = \frac{\dfrac{p_a - p_v}{\rho} - gh_s + \dfrac{C_{\bar{I}}^2}{2}}{E}$$

10   Assuming a draft tube outlet section of 25 m², compute the required Z_{ref} to achieve a Net Positive Suction Head (NPSH) of 11.9 m. [2 point]

This condition can be expressed as:

$$NPSH = H_{\bar{I}} - \frac{p_v}{g\rho} - Z_{ref} = 11.9m$$

The flow velocity at the draft tube outlet: $C_{\bar{I}} = \dfrac{Q_t}{A_{\bar{I}}} = \dfrac{0.99 \cdot 50m^3 \cdot s^{-1}}{25m^2} = 1.98m \cdot s^{-1}$

Therefore :

$$Z_{ref} = \frac{p_a - p_v}{\rho g} + Z_{\bar{B}} + \frac{C_{\bar{I}}^2}{2g} - NPSH = \frac{1 \times 10^5 - 2343}{998 \cdot 9.81} + 1025 + \frac{1.98^2}{2 \cdot 9.81} - 11.9 = 1023m$$

Pumped storage power plant – pump mode

Energetic efficiency	η_e	0.92	-
Volumetric efficiency	η_q	0.99	-
Global machine efficiency	η	0.90	-
Rated discharge per unit	Q	48	$m^3 s^{-1}$

Table 3 – Pump mode parameters of the power unit

Let's now move to study the power plant in pumping mode. The operation parameters are listed in Table 3. For this operating condition, it is assumed that the inlet velocity $\vec{C_{\bar{1}}}$ is axial and uniformly distributed. The outlet flow is also radially uniform. All the flow distribution coefficients k_{cu} and k_{cm} of global Euler equation, are then assumed equal to 1.

11   Compute the meridional component of the inlet absolute velocity $Cm_{\bar{1}e}$. [1 point]

To compute the meridional component of the absolute velocity at the pump inlet, we need to consider the discharge before the leakage losses:

$$Cm_{\bar{1}e} = \frac{Q_t}{A_{\bar{1}e}} = \frac{4 \cdot Q}{\pi D_{1e}^2 \cdot \eta_q} = \frac{4 \cdot 48}{\pi 2.8^2 \cdot 0.99} = 7.87 \, m \cdot s^{-1}$$

12   Compute the inlet tangential velocity $U_{\bar{1}e}$ and the relative inlet flow angle $\beta_{\bar{1}e}$. [1 point]

The rotational speed ω in pump mode is the same as in turbine mode since the grid frequency doesn't change: $U_{\bar{1}e} = \frac{\omega \cdot D_{\bar{1}e}}{2} = \frac{31.42 \cdot 2.8}{2} = 43.99 \, m \cdot s^{-1}$

And $\mathrm{tg}\left(\beta_{\bar{1}e}\right) = \frac{Cm_{\bar{1}e}}{U_{\bar{1}e}} \rightarrow \beta_{\bar{1}e} = \mathrm{tg}^{-1}\left(\frac{Cm_{\bar{1}e}}{U_{\bar{1}e}}\right) = 10.14°$

13   The hydraulic power in pump mode for this power plant corresponds to the 97% of the hydraulic power in turbine mode. Compute the specific energy. [2 point]

In turbine mode: $P_h = 100.09$ MW

Therefore, in pump mode: $P_h = 100.09$ MW $\cdot 0.97 = 97.09$ MW

The specific energy reads : $E = \frac{P_h}{\rho \cdot Q} = \frac{97.09}{998 \cdot 48} = 2026.8 \, \mathrm{J \cdot kg^{-1}}$

14   Compute the transferred specific energy. [1 point]

The transferred specific energy reads : $E_t = \frac{E}{\eta_e} = \frac{2026.8 \, \mathrm{J \cdot kg^{-1}}}{0.92} = 2203.0 \, \mathrm{J \cdot kg^{-1}}$

15   Compute the outlet tangential velocity U_{1e} and the tangential component of the absolute outlet velocity Cu_{1e}. [1.5 point]

Using Euler equation : $E_t = Cu_{1e}U_{1e} - Cu_{\bar{1}e}U_{\bar{1}e} = Cu_{1e}U_{1e}$ *since the inlet flow is axial, i.e.*
$Cu_{\bar{1}e} = 0$

$$U_{1e} = \frac{\omega \cdot D_{1e}}{2} = \frac{31.42 \cdot 3.5}{2} = 54.99 m \cdot s^{-1} \text{ and } Cu_{1e} = \frac{E_t}{U_{1e}} = \frac{2203.0}{54.99} = 40.06 m \cdot s^{-1}$$

16 Is this operating condition the one corresponding to the maximum power transfer condition? Justify your answer. [1.5 point]

No, the maximum power transfer condition is fulfilled when $Cu_{1e} = \frac{U_{1e}}{2}$.

In the current operating condition, we have $Cu_{1e} = 40.06 m \cdot s^{-1} > \frac{U_{1e}}{2}$

17 Assume an outlet blade angle $\beta_{1b} = 24°$ and no-slip condition fulfilled. For the same machine rotating velocity and the same efficiencies as listed in Table 3, provide the values of discharge Q and net head H that would allow the machine to operate in its maximum power transfer. [2.5 point]

The pump can operate at the maximum power if $Cu_{1e} = \frac{U_{1e}}{2} = \frac{\omega \cdot D_{1e}}{4}$. *For the same rotating*

velocity: $Cu_{1e} = \frac{U_{1e}}{2} = \frac{54.99 m \cdot s^{-1}}{2} = 27.50 m \cdot s^{-1}$

From Euler equation: $E_t = Cu_{1e} \cdot U_{1e} = \frac{U_{1e}^2}{2}$ *and since* $E_t = g\frac{H}{\eta_e}$ *the net head H can be*

deduced: $H = \frac{E_t \cdot \eta_e}{g} = \frac{U_{1e}^2 \cdot \eta_e}{2 \cdot g} = \frac{54.99^2 \cdot 0.92}{2 \cdot 9.81} = 141.79 m$, *which corresponds to a specific*

energy of $E = gH = 141.79 \cdot 9.81 = 1391 J \cdot kg^{-1}$

Moreover, at the maximum power condition, the following relation on Q_t holds:

$Q_t = \frac{U_{1e}A_{1e}tg(\beta_{1e})}{2}$.

Assuming no-slip condition:

$$\beta_1 = \beta_{1b} \rightarrow Q_t = \frac{U_{1e}A_{1e}tg(\beta_{1e})}{2} = \frac{U_{1e} \cdot \pi \cdot D_{1e} \cdot B \cdot tg(\beta_{1b})}{2} = \frac{54.99 \cdot \pi \cdot 3.5 \cdot 0.6 \cdot tg(24°)}{2} = 80.76 m^3 \cdot s^{-1}$$

And the optimal discharge can be deduced $Q = Q_t \cdot \eta_q = 80.76 \cdot 0.99 = 79.95 m^3 \cdot s^{-1}$

Now imagine that you are in charge of testing the reduced scale model of this pump-turbine in pumping mode, with a scale ratio $\lambda_{scale} = 4.80$. To ensure that the results you get from the tests are reliably representing the behavior of the real scale machine, you must correctly determine several operational parameters. To do that, consider the rated operating condition (and not the optimum computed in question 18)). The subscript $_M$ indicates a quantity at Model scale, whereas the subscript $_P$ indicates the real scale machine (i.e. the Prototype scale).

18 Compute the different IEC Factors for speed, discharge, torque and power, considering 1% of bearing power losses. Use D_{1e} as the reference diameter. [2 point]

With data of Table 1, the rotational speed $\omega = n \cdot 2\pi = -31.42 \, rad \cdot s^{-1}$ *and the torque computed*

as $T_m = \dfrac{P_h \cdot \eta_m}{\eta \cdot \omega} = \dfrac{\rho \cdot Q \cdot E \cdot \eta_m}{\eta \cdot \omega} = \dfrac{998 \cdot 48 \cdot 2026.8 \cdot 0.99}{0.9 \cdot 31.42} = 3.4 \times 10^6 \, N \, m$:

$$n_{ED} = \frac{nD_{1e}}{\sqrt{E}} = -0.3887 \quad Q_{ED} = \frac{Q}{D_{1e}^2 \sqrt{E}} = -0.0870 \quad T_{ED} = \frac{T_m}{\rho D_{1e}^3 E} = 0.0392$$

$$P_{ED} = \frac{P_m}{\rho D_{1e}^2 E^{1.5}} = -0.0958$$

19 Compute the pump inlet and outlet diameters of the reduced scale model $D_{\bar{1}e,M}$ and $D_{1e,M}$. [2 point]

With a scale ratio $\lambda_{scale} = 4.8$ *the diameters are scaled simply by:*

$$D_{\bar{1}e,M} = \frac{D_{\bar{1}e,P}}{\lambda_{scale}} = 0.583m \quad and \quad D_{1e,M} = \frac{D_{1e,P}}{\lambda_{scale}} = 0.729m$$

20 Determine the net head at reduced model scale H_M considering similarity of the Froude number between real scale and model scale. [1.5 point]

Froude similarity: $Fr_M = Fr_P$

$$Fr \sim \sqrt{\frac{H}{D}} \rightarrow H_M = \frac{H_P}{\lambda_{scale}} = \frac{2026.8}{9.81 \cdot 48} = 43.04m$$

21 Considering similarity of the IEC factors, compute the model scale rotational speed ω_M, discharge Q_M and mechanical torque $T_{m,M}$. [2 point]

Similarity of IEC factors: $n_{ED,M} = n_{ED,P}$, $Q_{ED,P} = Q_{ED,M}$ *and* $T_{ED,M} = T_{ED,P}$

Therefore, from question 19):

$$\omega_M = n_M \cdot 2\pi = \frac{-0.3887 \cdot \sqrt{E_M}}{D_{1e,M}} \cdot 2\pi = -68.84 \, rad \cdot s^{-1}$$

$Q_M = -0.087 \cdot \sqrt{E_M} \cdot D_{1e,M}^2 = -0.95 m^3 \cdot s^{-1}$ *(negative sign because in pump mode)*

$T_M = 0.0392 \cdot \rho \cdot E_M \cdot D_{1e,M}^3 = 6399 Nm$

Mechanical power at model scale: $P_M = T_M \cdot \omega_M = -440.6kW$ *(same if computed starting from similarity of* P_{ED} *factor)*